Average Bank Balance

An amount of money A_0 compounded continuously at interest rate r increases according to the law:

$$A(t) = A_0 e^{rt} \qquad (t=\text{time in years.})$$

a) What is the average amount of money in the bank over the course of T years?

b) Check your work by plugging in $A_0 = \$100$, $r = .05$ and $T = 1$; does the result seem plausible?

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a) $\text{Average} = \dfrac{1}{T-0} \displaystyle\int_0^T A(t)\, dt$

$= \dfrac{1}{T} \displaystyle\int_0^T A_0 e^{rt}\, dt$

$= \dfrac{A_0}{T} \left(\dfrac{e^{rt}}{r} \right)\Big|_0^T$

$= \dfrac{A_0}{Tr} \left(e^{Tr} - e^0 \right)$

$= \dfrac{A_0}{Tr} \left(e^{Tr} - 1 \right)$

b) $\text{Average} = \dfrac{100}{1(0.05)} \left(e^{1(0.05)} - 1 \right)$

$= 2000 \, (0.05127)$

$= 102.54$

$\therefore$ A simple interest over a year would yield $100(1+0.05) = 105$, so it is reasonable that the average would be 102.54.